A Fast Algorithm for Nonlinear QSM Reconstruction with Variational Penalties

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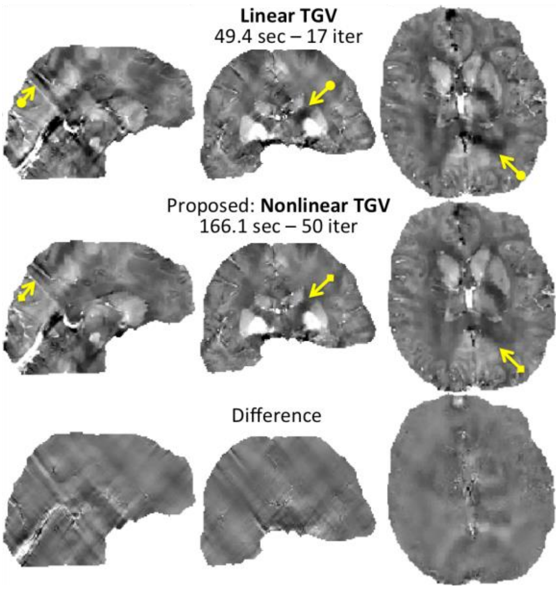
INTRODUCTION: Quantitative Susceptibility Mapping (QSM) involves the inversion of an ill-posed problem to derive tissue susceptibilities from the GRE phase. QSM inversion can be achieved through the minimization of a functional consisting of regularization and data fidelity terms. It has recently been shown that total generalized variation (TGV) penalty is preferable to TV regularization for QSM [1,2,3]. On the other hand, the data fidelity term comprises a susceptibility-to-field relationship based on the phase noise distribution. In most scenarios, such distribution can be approximated by a Gaussian function. However, such an assumption breaks down when SNR is low [4]. To address this issue, Liu et al. [5] proposed a data fidelity term based on a phase projection into the image domain, where the noise distribution becomes a complex Gaussian. An exhaustive comparison of noise models [6] showed the benefits of using such non-linear formulation, i.e. streaking artefact reduction, improved noise mitigation and overall more accurate susceptibility estimation. Here, we present a novel solution to the non-linear data fidelity term in conjunction with the TGV regularizer using the ADMM solver [3,7,8], which leads to close-form updates that accelerate the reconstruction, achieving computation speed comparable to the linear fidelity formulation while better mitigating dipole artifacts.

METHODS: We first performed variable splitting by introducing an auxiliary variable $z = DFx$ to decouple the equation system. Then, we formed the augmented Lagrangian functional, $argmin_{x,v,z} \frac{1}{2}\left\|w\left(e^{iF^Hz} - e^{i\phi}\right)\right\|_2^2 + \frac{1}{2}u \cdot \|DFx - z + s\|_2^2 + TGV(x,v)$, where ***s*** is the associated Lagrangian multiplier, ***w*** is a magnitude dependent weight, and ***u*** is a Lagrangian weight; followed by deriving the algorithm within the ADMM framework. The subproblem for χ and v was solved using closed-form solutions described in [3]. Regarding the **z** subproblem, we obtained the gradient of this functional in a voxel-wise decoupled structure: $f(F^Hz) = w^2 sin(F^Hz - \phi) + u \cdot (F^Hz - F^HDFx - F^Hs)$. To solve this nonlinear equation, we discretized the range of F^Hz in as few as four values, and evaluated the functional at each voxel. We used the value of **z** that minimizes the functional in this global search to initialize a Newton-Raphson local solver, looking for $f(F^Hz) = 0$. As such, the algorithm can effectively avoid being trapped into local minima.



The proposed solver was compared using a synthetic brain phantom (not shown) and 3T MRI *in vivo* data (Siemens Tim Trio, 32 channel head array, 1 mm^3 isotropic resolution, 240×192×120 mtx, TE/TR = 24.8/35 ms, T_{acq}=13.5min). Phase unwrapping and background subtraction were performed with Laplacian [7] and Laplacian Boundary Value (LBV) [8] methods, respectively. Results were evaluated qualitatively in terms of artifact prevention and noise management. RMSE relative to ground truth susceptibilities were also calculated for synthetic data.

RESULTS: They confirmed a qualitative improvement over the traditional TGV-QSM solver. Phantom based RMSE was approximately halved (not shown), yielding no streaking artifacts and improved noise properties. Notably, noise suppression was also achieved in zero-magnitude areas. Similarly, *in vivo* results also revealed an overall improvement in noise and artifact management (Fig. 1).

DISCUSSION AND CONCLUSION: The proposed non-linear solver signifies an improvement over linear TGV in terms of noise modeling and preventing streaking artifacts emanating from zero or low SNR regions. In practice, QSM in those regions can only be consistent with the aid of the regularization constraint. In terms of computational cost, although the present approach required greater memory usage, calculation times were comparable to those for the linear solver (<15% time increase per iteration). Notably, although a TGV regularizer was used here, the same algorithm structure could be used for other variational penalties. In addition, the proposed algorithm applied here to the background filtered phase could be extended to allow single-step processing directly from the raw GRE phase.

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